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Treasury Inflation-Protection Securities: Opportunities and Risks

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Summary

- The current inflation risk premium in U.S. 10-year yields is likely to be in the range of 21 to 35 bp.
- The appropriate measure of risk for TIPS is the duration with respect to real yields.
- The U.K. experience indicates that real yields correlate with economic growth rates while nominal to real yield spread depends on expectations and volatility of inflation.
- In the current economic climate, the relative volatility of 10-year real to nominal yields is expected to be about 80% and the nominal to real yield spread is expected to be uncorrelated with real yields.
- Inflation delay and seasonality of inflation affect the valuation of Treasury Inflation-Protected Securities (TIPS).
- Relative value of TIPS depends on investor views of future inflation.
- Ten-year TIPS can improve the risk-return profile of a portfolio of nominal fixed income securities even when they trade at a yield give-up to nominal securities.
- TIPS can be a more efficient hedge against declines in equity valuation than nominal Treasuries.

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I. INTRODUCTION

Issuance of inflation-indexed securities by the U.S. Treasury marks the first issue of inflation protection securities in the United States. There are currently no easy benchmarks to project the performance of these securities relative to other fixed income instruments. However, several foreign governments (Australia, Canada, Israel, Sweden, and the United Kingdom) have issued inflation-indexed securities for several years. In this report, we examine the United Kingdom's experience since 1985 to draw inferences about the likely behavior of U.S. inflation-indexed securities.

The U.K. economy comes closest to the U.S. economy in structural features. In particular, monetary and fiscal policies in the U.K. over the last couple of years have pursued objectives similar to those in the U.S., making the U.K. experience relevant for the U.S. We use the U.K. experience to understand the relationships between economic regimes and real and nominal yields. Adjusting these relationships to reflect current U.S. economic circumstances allows us to project inflation risk premium, relative price performance, and hedge ratios for the new inflation-indexed securities. We then evaluate the attractiveness of these securities for U.S. investors. Our analysis is focused on the 10-year sector where initial issuance is likely to be concentrated.

II. SECURITIES STRUCTURE

The Inflation Index

The new U.S. inflation-protection notes will be indexed to the non-seasonally adjusted U.S. City Average All Items Consumer Price Index for All Urban Consumers (CPI-U). This index compares the cost in a given month to a base month for purchasing a fixed market basket of goods and services at market prices. The basket consists of total expenditures on items such as food, clothing, shelter, transportation, medical services, and other goods and services necessary for day-to-day living.

Principal Adjusted for Inflation

The securities are indexed to inflation by adjusting the principal outstanding per unit of the par amount. The adjustment is in the ratio of the value of the CPI-U applicable at a given date to the CPI-U applicable at issuance. The coupon rate on the securities is fixed. Semiannual coupon payments for a given par amount are determined by multiplying the inflation-adjusted principal by the fixed coupon rate. At maturity, investors receive the greater of the inflation-adjusted principal or the par amount; i.e., investors have a put option on the principal against the possibility of deflation over the life of the security. This put option, however, applies only to the principal amount. Coupon payments are calculated using the inflation-adjusted principal at all times, including at maturity.

Inflation Is Reflected with a Two-month Delay

For calculating coupon and principal payments, the CPI-U is used with a two-month lag. Specifically, the reference index on the first calendar day of a month is the CPI-U for the third preceding calendar month. For example, the reference index applicable to April 1 would be the CPI-U for the month of January (released in February). Similarly, the reference index for May 1 would be the CPI-U for February (released in March). If the index is revised in a subsequent month, the unrevised value of the index will continue to be used as the reference value. The reference index value for any date within a month is then calculated from the values of the index at the beginning of the current and the subsequent month, using linear interpolation based on calendar dates. Since the CPI-U for each month is released in the early part of the following month, this scheme allows the value of the reference index to be known at the beginning of the upcoming as well as the beginning of the subsequent month at least two weeks before the month begins. The delay with which CPI-U affects the cash flows of the security has implications for effective real yield, price volatility, and response to changes in nominal rates. We discuss these effects in the following section.

Securities Eligible for Stripping

The securities will be eligible for stripping into interest and principal components. Unlike fixed principal securities, however, the interest components of different inflation-protection securities, at least initially, will not be fungible even though they may have the same maturity. This is because each security may have a different base value of the index at issuance. Payments on the stripped interest components will, therefore, depend on the security that was stripped. The Treasury is currently working on a mechanism for making the interest-only components fungible.

III. VALUATION BASICS

We discuss the valuation of these securities under the assumption that real yield curve is known and describe the calculation of durations and convexities with respect to real yields. Then we analyze the effect of the put option on the principal, the lag with which inflation is reflected in cash flows, and the choice of inflation index.

Real Yields Are Similar to Nominal Yields

The concept of real yield is similar to the more familiar nominal yield, with the difference that future cash flows are measured in terms of current purchasing power. When the cash flows of a security are fixed in nominal dollars, the yield (or internal rate of return) can be expressed in nominal terms because the nominal cash flows are known. Alternatively, future cash flows could be expressed in terms of current purchasing power by deflating them using future inflation. A real yield could then be calculated using these real future cash flows. Since future inflation is unknown the realized real yield on nominal securities is uncertain while the realized nominal yield is certain (ignoring reinvestment risk). Nominal yield is a convenient measure for comparing different nominal securities or the same security over time.

The situation is reversed for inflation-indexed securities. Future cash flows are certain in terms of current purchasing power while nominal cash flows are uncertain. Consequently, the realized real yield to maturity of these securities is certain but the realized nominal yield is uncertain, making real yield a natural measure for comparing a security to other real securities or to itself over time. The computation of real yield for an inflation-indexed security is straightforward and identical to the calculation of the nominal yield of a nominal security, except that real cash flows are substituted for nominal cash flows. The real yield can be used to compare inflation-indexed securities, including those of issuers other than the U.S. Treasury.

While nominal yields of real securities and real yields of nominal securities could be projected assuming a particular course of future inflation, we think static yields based on such projections would be of limited value in guiding investment decisions. The fundamental difference between the two securities arises because future inflation is uncertain. If inflation were certain, not only would nominal yields of inflation-indexed securities and real yields of nominal securities be certain and easily calculated, but the market level of nominal and real yields would also be different. In later sections of this report, we suggest how investors could make relative value comparisons of inflation-indexed and nominal securities.

Durations and Convexities Can Be Calculated for Real Yields

Duration and convexity for a nominal security are calculated with respect to nominal yield. Similarly, duration and convexity for an inflation-indexed security can be calculated with respect to real yield. The formulas for the two cases are the same with the real yield substituted for the nominal yield. In Figure 1, we compare durations and convexities for inflation-indexed securities with respect to real yields and maturity-matched nominal securities for nominal yields. Both durations and convexities of inflation-indexed securities are greater than those for maturity-matched nominal securities. This arises because the durations and convexities of inflation-indexed securities are calculated for real yield, which is approximately 300 basis points (bp) lower than the yield of nominal securities. Because of the lower rate of discounting, the longer term cash flows of inflation-indexed securities contribute a greater share of present value, making durations and convexities larger. This is the same reason durations and convexities of nominal securities increase with a decline in yield.

The durations and convexities of inflation-indexed securities are not directly comparable to those of nominal securities. The performance of the two types of securities will depend on the relative behavior of real and nominal yields. There could be substantial basis risk between real and nominal securities. Duration of real securities with respect to nominal yields (i.e., their hedge ratios) is, therefore, likely to vary depending on expectations of economic fundamentals.

Figure 1. **Durations and Convexities of Inflation-indexed and Nominal Coupon Securities**

	Inflation-indexed (at 3.5% real yield)		Nominal (at mkt. yld. on 12/27/96)	
Maturity	**Duration**	**Convexity**	**Duration**	**Convexity**
5-year	4.55 yr	23.96 yr^2	4.26 yr	21.70 yr^2
10-year	8.38	81.70	7.10	63.89
30-year	18.48	461.16	12.94	267.28

IV. MODIFICATIONS TO THE VALUATION FRAMEWORK

The pricing framework for Treasury Inflation-Protected Securities (TIPS) needs to account for the following three features: the delay with which inflation affects cash flows; seasonality in the inflation index; and the put option on the principal payment at maturity.

Impact of Inflation Delay

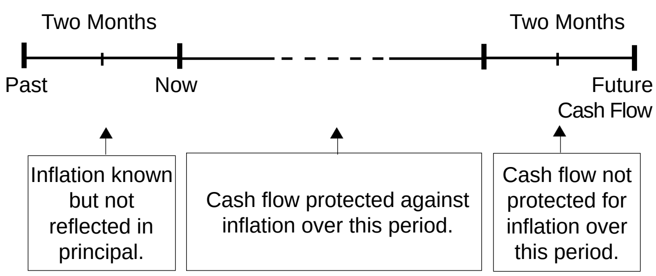
Inflation Delay Affects the Yield of TIPS . . .

For any calendar date, the index that determines the principal adjustment of inflation-indexed securities is based on the CPI-U of two months ago. For example, the index value for April 1 is the CPI-U for January, and during April the index adjustment is made assuming that inflation in February occurred at a uniform rate throughout the month. This means that the index value for any date in April also reflects inflation two months earlier (Figure 2).

The Current Inflation Effect: On any date, market participants will know the extent of inflation that is not reflected in the principal of the security. If this fact is ignored, the effective real yield would be miscalculated. For instance, on March 31, the principal is adjusted for the inflation experience up to January 31. However, market participants are aware of the inflation during February and March[1] and know it will be reflected in principal adjustments in the future. Investors who fail to make this adjustment would be underpricing these securities for a given real yield.

[1] We ignore for the moment the fact that inflation is measured on a monthly basis and is announced with a lag after month-end (typically within five to seven calendar days).

Figure 2. **Two Impacts of the Inflation Delay on Effective Real Yield**



The Nominal Yield Effect: The second effect is that all future cash flows are unprotected from inflation, which occurs in the two-month period leading up to the date of payment when the index value for computing the amount of the payment is frozen. This means that in evaluating the present value of future cash flow, the cash flow should be discounted for a two-month period by a rate that reflects its exposure to inflation risk over that period, and for the remainder of the period by a rate that reflects its protection against inflation. In other words, the rate of discount for the final two months should be the two-month forward rate derived from the nominal yield curve, and discounting for the balance of the period should be at the real yield. With nominal forward rates greater than real yields, the effective real yield will be lower at a given price and the price lower for a given real yield. (See Appendix A for the formula to calculate effective real yields taking into account the delay).

An alternative method of accounting for the inflation delay is sometimes used in the U.K. markets, where the securities have an eight-month delay. That method calculates a breakeven rate of future inflation at which the real yield of the security plus the breakeven inflation rate equals the nominal yield of a maturity-matched nominal security. The breakeven inflation rate is used to calculate future cash flow for the inflation-indexed security. We think this method has little theoretical justification.

. . . And Results in Price Sensitivity to Current Inflation and Nominal Yields

TIPS may experience price volatility due to uncertainty in current inflation. Inflation is announced on a monthly basis. As market expectations of current inflation change, the price of the securities may react to the updating of the inflation adjustment of the principal for the two-month delay. Our method of discounting each cash flow for two months at nominal forward rates also leads to duration of these securities with respect to nominal yields. This sensitivity arises even if real yields are constant, and is distinct from and incremental to the price sensitivity that may arise due to a correlation between real and nominal yields. From a portfolio perspective, this price sensitivity is not likely to be significant.

Our Estimates of the Delay Effect

Figure 3 shows estimates of the various effects of the delay for 5-, 10-, and 30-year inflation-indexed securities. For the 10-year maturity, inflation (assumed at 2.5% per year) over the immediate past two months would

Figure 3. **Estimating the Impact of Inflation Delay for Coupon Securities**
Flat real yield curve of 3.5%, Nominal yield curve as of 12/27/96

Maturity	Change in Effective Real Yield Due to (in bp): Past Inflation @ 2.5% p.a.	Future Unaveraged Inflation Risk	Total	Duration with Respect to Nominal Yields	Price Sensitivity with Respect to 1% p.a. Change in Current Inflation
5-year	9.1	-10.9	-1.8	0.16 yr	0.16%
10-year	4.9	-6.4	-1.5	0.16	0.16
30-year	2.2	-2.5	-0.3	0.16	0.16

increase the effective real yield by 4.9 bp. Discounting future cash flows by nominal rates for the two-month unprotected period would lower the effective real yield by 6.4 bp. The net effect is a decrease in the real yield of 1.5 bp. Although the net effect is small, the individual effects are relatively large. Under different economic scenarios, the net effect could be 5-6 bp for the 10-year. The effect is greater for shorter maturities and less for longer maturities. For all maturities, the duration related to nominal rates due to the delay is approximately two months (0.16 years). The price sensitivity related to a 1% increase in inflation due to the delay is estimated to be 0.16%.

Seasonality in the Index Could Have a Significant Price Impact

Use of the non-seasonally adjusted CPI-U for calculating the principal of the securities is a potential source of price volatility. Figure 4 shows the average and range, for the period 1989-1996, of the unannualized difference between the non-seasonally and seasonally adjusted CPI-U series by month. The non-seasonal index has always been higher in the first quarter and lower in the fourth quarter. For example, for the month of January the difference has ranged between 0.07% and 0.22% with an average of 0.17%. In December the range has been -0.37% to -0.20% with an average of -0.24%. Seasonality affects valuation through its impact on the current (unannounced) inflation; therefore, investors must account for seasonal variations to make rational conjectures about current inflation.

Figure 5 shows the estimated potential price impact of this seasonality effect. We assumed a required real yield of 3.5%. We calculated, for a 10-year inflation-indexed note, the difference in price assuming adjustment for the average seasonality effect over a two-month period and ignoring it. We did this at the end of each month during the year. For example, the price difference reported for the month of January is for the combined seasonality effect for December and January. Our estimates show that prices could be affected by as much as 11/32 in either direction due to seasonality. This may be an overestimate because it assumes that investors completely ignore the seasonality effect, but it illustrates the need to account for seasonality. Also, variability in the seasonality effect is likely to induce unavoidable price volatility in these securities.

Figure 4. **Seasonality in Inflation, 1989-1996**
Unannualized difference between non-seasonally and seasonally adjusted CPI-U

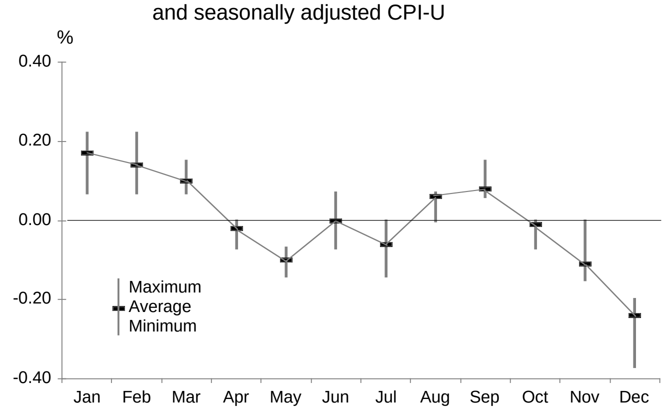
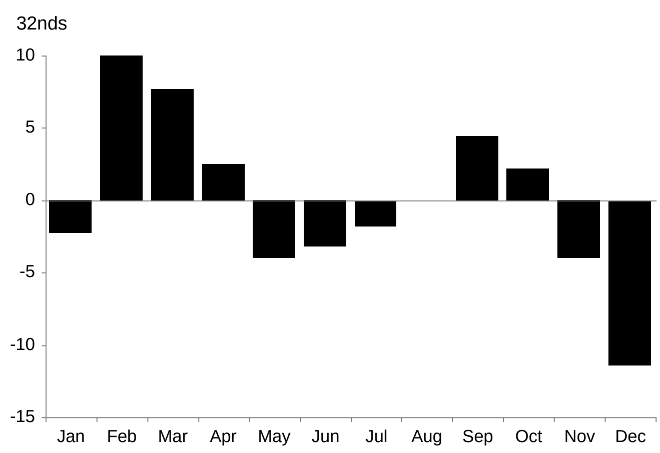


Figure 5. **Potential Price Impact of Seasonality in Inflation**
Difference in price of 10-year inflation-indexed note with and without seasonal pattern at a 3.5% real yield



The Put Option Is of Little Value

Investors will receive at maturity a principal amount that is the greater of the inflation-adjusted principal or its par amount. It can be valued as an option to exchange an inflation-adjusted zero coupon bond for a nominal zero coupon bond with a par amount equal to the par amount of the inflation-indexed zero. The option provides a floor for inflation-indexed zero coupons in that their price cannot fall below the price of a maturity-matched nominal zero coupon. At current market levels the price of a real zero coupon would be approximately 140% of the price of a 10-year nominal zero. The value of the floor provided by the prices of nominal zeros is, therefore, not meaningful.

V. RELATION BETWEEN NOMINAL AND REAL YIELD CURVES

Expected Real Return of Nominal Bonds Includes Risk Premium for Inflation . . .

The nominal and real yield curves can be best compared in terms of zero coupon bonds. A nominal zero coupon bond offers a certain nominal pay-off at maturity. The purchasing power at maturity of this pay-off is, however, uncertain. An inflation-indexed zero coupon bond with the same maturity offers a pay-off that is protected against inflation. The purchasing power of this pay-off is certain. If investments are considered in terms of purchasing a basket of goods and services at a given maturity and not in dollar terms, an inflation-indexed zero coupon bond is a risk-free asset and a nominal zero coupon bond is a risky asset. Both the zero coupons have no reinvestment risk. In terms of bond equivalent yields, for maturity-matched nominal and real zero coupons,

$$(1+Y_n/2) \times ([\text{Expected value } (I_{today}/I_{mat})]^{\frac{1}{2T}}) = [(1+Y_r/2)(1+r/2)] \qquad (1)$$

where

- T = Maturity, in years, of zero coupons;
- Y_n = Yield of nominal zero coupon;
- Y_r = Yield of real zero coupon;
- I_{mat} = Uncertain value of inflation index at maturity;
- I_{today} = Known value of inflation index today;
- r = Risk premium, in bond equivalent terms, for bearing inflation risk in the nominal zero coupon over its maturity.

The term on the left side of the equation is the expected real yield of the nominal zero coupon bond. This equals the risk-free real yield of the inflation-indexed zero coupon, grossed-up by the risk premium. The term (I_{today}/I_{mat}) is the reciprocal of total inflation over the maturity of the bonds. The expected value of this term is the expected reduction in purchasing power to be applied to the pay-off of the nominal zero coupon bond to determine its expected real pay-off.

. . . Adjusted for the Inflation Convexity Effect

It is usual to express the relationship between nominal and real yields in terms of real yields, expected inflation, and risk premium. Care must be exercised, however, in translating Equation (1) into this form. When inflation is uncertain, the expected value of the reciprocal of inflation does not equal the reciprocal of the compounded value of expected annual inflation. For example, if average inflation over the next 10 years is expected to be 3%, the expected value of the reciprocal of inflation will not be $1/(1.03)^{10}$. This effect arises because of Jensen's Inequality[2] and is called the convexity effect. A translation of Equation (1) that takes the convexity effect into account is

$$(1+Y_n/2) = [(1+Y_r/2)(1+I/2)][(1+r/2)/(1+c/2)] \qquad (2)$$

where

$1+I/2 = [\text{Expected value } (I_{mat}/I_{today})]^{\frac{1}{2T}}$ is the expected value of inflation based on semiannual compounding;

$1+c/2$ = Convexity effect.

The magnitude of the convexity effect depends on the volatility of future inflation. Figure 6 shows the magnitude of the convexity effect for a range of volatilities of future inflation. We computed the convexity effect by simulating inflation over a 10-year horizon using a mean-reverting inflation process. We varied the degree of mean reversion to change the volatility of the inflation process. Our simulations show that the convexity effect is only 1 bp if future inflation is as stable as its recent history, which corresponds to annualized standard deviation of inflation of 0.38%. However, if inflation were to become more volatile, the convexity effect would increase roughly in proportion to the variance of the inflation process. For example, the convexity effect would be 12 bp for annualized standard deviation of inflation of 1.10%.

[2] Jensen's Inequality states that Expected value $[f(x)] \neq f(\text{Expected value } [x])$ if $f(\cdot)$ is not a linear function.

Figure 6. **Convexity Effect in Calculation of Expected Real Return of 10-year Zero Coupon Bonds**

Annualized Standard Deviation of Inflation over 10 years (%)	0.38	0.68	1.10	1.88
Annualized Convexity Effect (bp)	1	4	12	34

In Equation (2), the risk premium and the convexity effect influence the relationship between nominal and real yields in opposing directions. Because both are proportional to the variance of future inflation, the risk premium and the convexity effect combined can be thought of as the net risk premium for a given maturity point on the yield curve. If, however, different points on the yield curve or risk premia in different markets are to be compared, the convexity effect needs to be accounted for.

VI. THE U.K. EXPERIENCE

Estimating the Real Yield Curve Is Not Straightforward

Estimating the real yield curve from the nominal yield curve requires estimating market expectations of inflation as well as the risk premium and the convexity effect. None of these quantities is directly observable in the U.S. markets and all are likely to change over time. Estimating these quantities from economic fundamentals is not straightforward. Expected inflation depends on the market's expectations of future monetary policy. The risk premium and the convexity effect depend on market estimates of volatility of future inflation.

Inflation-linked Gilts Offer Help

An inflation-linked Gilts market has existed in the U.K. since 1985. Currently, inflation-linked Gilts amount to approximately 20% of total Gilts in market value terms. A series of bonds is outstanding, ranging in remaining maturities from 1 year to 28 years. Inflation-linked Gilts are less liquid than nominal Gilts with approximately three times as wide bid-ask spreads. Data are available on this market since 1985, and although the data for earlier periods may not be as accurate as for later periods, we think it is adequate for our purposes. The period since 1985 covers a wide range of monetary and fiscal regimes in the U.K., and the history of inflation-linked Gilts over this time offers a look at the relationships between real yields, nominal yields, and the state of the U.K. economy. The U.K. authorities have followed a stable inflation policy over the last couple of years that is similar to the current U.S. monetary regime. Inferences drawn from the last couple of years are, therefore, likely to be most relevant to potential U.S. experience. High inflation periods in the past, on the other hand, can say something about the type of inflation insurance inflation-indexed securities are likely to provide.

We constructed a constant maturity 10-year weekly real yield series from the prices of inflation-linked Gilts. These Gilts are indexed to the U.K. Retail Price Index (RPI) with a delay of eight months. To discount the cash flows of indexed-linked Gilts for the period of uncovered inflation risk due to the delay, we used forward rates derived from the U.K. yield curve splines, estimated by Lehman Brothers. (Our yields, therefore, differ from those derived using the breakeven inflation method employed in the U.K.) Our constant maturity series was created by linear interpolation of real yields for the two closest maturity inflation-linked Gilts. We also constructed a constant maturity 10-year nominal yield from our splines for the U.K. yield curve. The spread between 10-year nominal and real yields consists of inflation expectations and the effective risk premium (i.e., combined inflation risk premium and convexity effect).

The U.K. Has Experienced Varied Economic Regimes

Figure 7 summarizes monetary regimes in the U.K. since 1985. Broadly speaking, the period can be divided into three regimes. The period up to September 1990 is characterized by relatively tight monetary policy, high base lending rates, and high and volatile inflation. October 1990 through August 1994 was a transition period from high to lower inflation accompanied by a lowering of the base lending rate by almost 10.00%. The transition can be subdivided into two phases. The first period up to August 1992 was characterized by easing constrained by the need to maintain the sterling in the Exchange Rate Mechanism (ERM). During this period, inflation dropped to a rate of 3.81% per year but real GDP contracted by an annualized rate of 1.05% even as the base lending rate was lowered by 5.00%. During the second part of the transition from September 1992, when the sterling fell out of the ERM, to August 1994, the base lending rate was reduced by a further 4.75%, inflation fell to a 2.07% annual rate, and real GDP grew at a 3.13% annual rate. The period from September 1994 through November 1996 is characterized by

Figure 7. **History of Economic Regimes in the U.K.**

		Base Lending Rate (%)		Chng. in (Ann. %)	
Period	**Monetary Policy**	**Start**	**End**	**RPI**	**GDP**
1/85-7/85	Tight but easing	14.00	11.50	8.35	7.28
8/85-2/87	Tight but stable	11.50	11.00	3.40	5.97
3/87-5/88	Ease	11.00	7.50	4.60	4.13
6/88-10/89	Tighten	7.50	15.00	7.40	1.59
11/89-9/90	Tight	15.00	15.00	11.00	0.14
10/90-8/92	Ease constrained by ERM	15.00	10.00	3.81	-1.05
9/92-8/94	Unconstrained ease	10.00	5.25	2.07	3.13
9/94-4/95	Moderate tightening	5.25	6.75	4.49	7.14
5/95-11/96	Moderate easing	6.75	6.00	2.14	0.59

relatively stable monetary policy with the base lending rate in a range of 1.50% and inflation averaging around 3%. This last period, particularly since May 1995, is similar to the current monetary regime in the U.S.

Real Yields Have Been Less Volatile than the Yield Spread

Histories of 10-year real yields and the nominal to real yield spread are shown in Figures 8 and 9, respectively. Real yields in the U.K. have varied between 2.62% and 4.98% over the last 11 years. Most of this variability occurred around the breakdown of the ERM. Real yields reached a high just before sterling fell out of the ERM. The low came in early 1994 as the U.K. economy adjusted after the constraining influence of ERM was removed. The yield spread has been more volatile than real yields, with a range of 3.52% to 8.66%. The highs came in mid-1990 when the U.K. was experiencing 11% inflation. The lows occurred in late 1993 and early 1994 when inflation was at its lowest.

Figure 10 shows the relative volatilities of 10-year real yields and the nominal to real yield spread. For the period January 1985 to November 1996, the yield spread range was 514 bp while the real yield range was 236 bp or 46% of the range of yield spread. Between May 1995 and November 1996, the yield spread range was 91 bp while the real yield range was 45 bp or 49% of the spread range. A comparison of standard deviation of weekly yield changes shows that real yields were 57% as volatile as the yield spread for the entire period as well as the last subperiod. This pattern of relative volatilities was, with one exception, also typical of the other subperiods with real volatilities ranging between 39% and 64% of the volatility of the yield spread. The exception came following the ERM breakdown when real yields were 85% as volatile as the yield spread. For projecting the relative volatilities of real yields versus the nominal to real yield spread for the U.S., we used 57% as the benchmark for the U.K.

Real Yields Are Related to Economic Growth

The level of real yields in the U.K. has been related to economic growth rates. Figure 8 suggests that in several subperiods since 1985 the average level of real yields has been constant, although the average has differed across periods (the differences are statistically significant). The periods of stable average real yields have some correspondence to the economic regimes identified in Figure 7. Furthermore, a relationship exists between the average level of real yields and economic growth. Figure 11 shows the average level of real yields

Figure 8. **10-year Real Yields in the U.K.,** 1985-1996

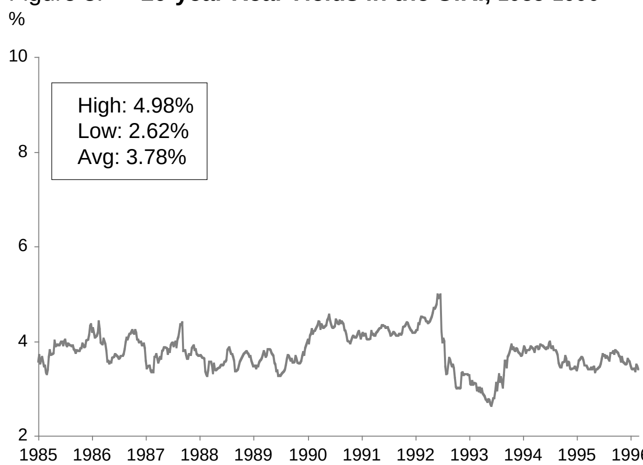


Figure 9. **Difference between Nominal and Real 10-year Yields in the U.K.,** 1985-1996

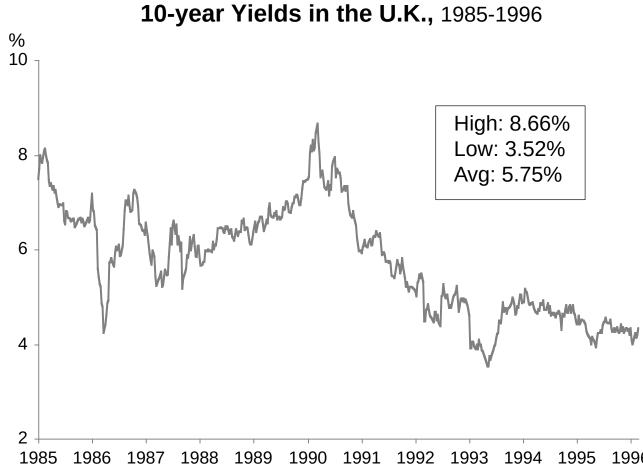
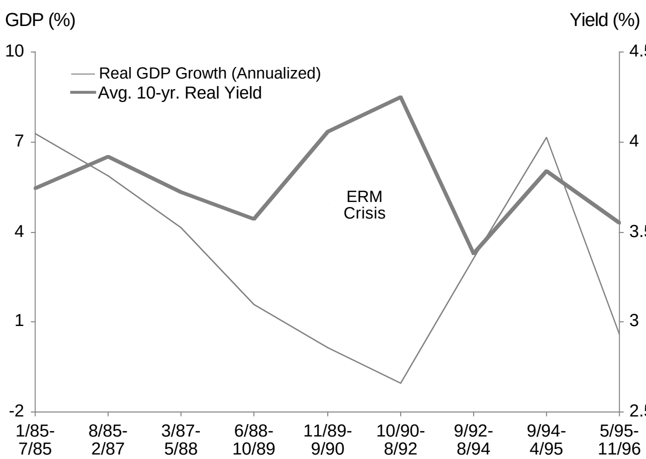


Figure 10. **Volatilities of Real Yields and Nominal to Real Yield Spread for the U.K. 10-year Sector,** Weekly data

	1/85-11/96	5/95-11/96
1. Range of Real Yields (bp)	236	45
2. Range of Nominal to Real Yield Spread (bp)	514	91
3. Ratio [(1) / (2)]	0.49	0.49
4. Annualized Standard Deviation of Weekly Changes in Real Yields (bp)	69	47
5. Annualized Standard Deviation of Weekly Changes in Nominal to Real Yield Spread (bp)	122	82
6. Ratio [(4) / (5)]	0.57	0.57

Figure 11. **GDP Growth and 10-year Real Yields in the U.K.**



along with real GDP growth for each of the subperiods. Except for the period from November 1989 to August 1992, higher average real rates have generally been associated with higher GDP growth. The variation in average real yields is less than the variation in GDP growth. The relationship accords with economic intuition: a growing real economy creates real investment opportunities and demand for capital, which results in higher real rates.

The period from November 1989 to September 1992 is the exception, with the highest level of real yields but the lowest rate of GDP growth. We think this is related to the reunification of Germany, which created substantial demand for capital resulting in high real rates. With the sterling locked into the ERM, the high real rates were exported to the U.K. The U.K. economy, however, was in a recession because of tight monetary policy. This disequilibrium between the capital markets and the U.K. economy was unsustainable and led ultimately to the sterling falling out of the ERM.

The Yield Spread Is Related to Inflation

While real yields in the U.K. have been related to the economic growth rate, the nominal to real yield spread has been related to inflation. Figure 12 shows the annualized inflation rate and the average yield spread for each of the U.K. monetary regimes. The average yield spread was highest from November 1989 to September 1990, when inflation in the U.K. was also highest. Other high inflation periods, such as January to July 1985 and June 1988 to October 1989, also correspond to high yield spreads. Conversely, the lowest yield spreads occurred from September 1992 to August 1994 and May 1995 to November 1996, when inflation was lowest.

Estimation of Inflation Risk Premium

We used the nominal to real yield spread to estimate the effective risk premium (i.e., risk premium plus convexity effect) in the U.K. To estimate expected inflation, we estimated an autoregressive process for inflation. Our process accounts for seasonality and shifts in inflation regimes: a high inflation regime to September 1990 (average inflation above 6%), a medium inflation regime from October 1990 to August 1994 (average inflation around 4%), and a low inflation regime from September 1994 to November 1996 (average inflation around 2.5%). Then for each month, we forecast the monthly inflation vector for 10 years. The forecast used the estimated parameters of the inflation process and the actual inflation history up to that month. The monthly vectors were used to compute an average inflation forecast for the following 10 years. We took this to be the market's expectation of inflation, which we subtracted from the yield spread to estimate the effective risk premium.

Figure 13 shows the average effective risk premium for each of the U.K. monetary regimes. We have less confidence in estimates for the period prior to October 1990 when inflation was high and volatile. Focusing on the period since October 1990, the average effective risk premium has ranged between 166 bp and 243 bp; from May 1995 to November 1996, the average was 197 bp.

Figure 12. **Inflation and 10-year Nominal to Real Yield Spread in the U.K.**

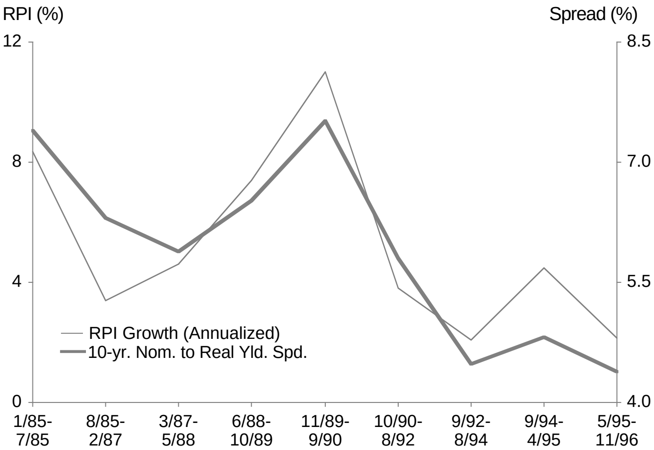


Figure 13. **Estimates of Inflation Risk Premium Plus Inflation Convexity Effect for the U.K. 10-year Sector**
Monthly data, 1985-1996

Period	**Avg. Risk Premium and Convexity Effect*** (bp)
1/85-7/85	137
8/85-2/87	125
3/87-5/88	-28
6/88-10/89	29
11/89-9/90	129
10/90-8/92	166
9/92-8/94	211
9/94-4/95	243
5/95-11/96	197

* Equals Nominal yield - Real yield - Expected inflation.

Since market expectations of inflation are not observable, any separation of the observed spread into expected inflation and risk premium may be viewed as arbitrary. For example, we estimate the average effective risk premium from September 1994 to April 1995 at 243 bp. This is a conservative estimate and does not capture increases in expected inflation as the Bank of England was tightening. Similarly, the current low inflationary environment in the U.K. could result in our underestimating long-term expected inflation and consequently overestimating the effective risk premium.

The Fisher Hypothesis Is Mostly Validated

An important factor determining the relationship between real and nominal yields is the correlation between real yields and the nominal to real yield spread. The often cited Fisher hypothesis[3] states that in a stochastic setting, the real rate and the spread (which consists of inflation expectations and the effective risk premium) are independent. The existence of inflation-linked Gilts offers an opportunity to test the Fisher hypothesis for long-term yields. These correlations for the U.K. 10-year sector are shown in Figure 14. The correlations were statistically significant in three of the nine subperiods, and all the significant correlations were negative. Thus, the Fisher hypothesis held for most of the period for 10-year yields. When it did not hold, the correlations were always negative. Further, in all three periods when the correlations were significant and negative, the Bank of England was aggressively easing monetary policy. Thus it appears that in the U.K., aggressive monetary easing leads to a lowering of expected future inflation (assuming the risk premium is not correlated with real yields) and an increase in real yields which, in turn, is consistent with an expected increase in economic growth rates. The converse does not hold with equal force. When monetary policy was being tightened aggressively from June 1988 to October 1989, the correlation was negative but not statistically significant. When changes in monetary policy were moderate, as in the period from September 1994 to November 1996, correlations between changes in real yield and yield spread were zero.

[3]Irving Fisher, *The Theory of Interest* (New York: Macmillan), 1930.

Figure 14. **Correlation between Changes in 10-year Real Yields and Nominal to Real Yield Spread for the U.K.**
Weekly data, 1985-1996

Period	**Correlations**
1/85-11/96	-0.167*
1/85-7/85	-0.599*
8/85-2/87	-0.132
3/87-5/88	-0.248*
6/88-10/89	-0.105
11/89-9/90	0.020
10/90-8/92	-0.133
9/92-8/94	-0.212*
9/94-4/95	-0.035
5/95-11/96	0.056

* Statistically significant at 5% level.

VII. ESTIMATES OF THE INFLATION RISK PREMIUM IN THE U.S. MARKET

Market Risk Premium Can Help in Evaluating TIPS

Nominal and real yields are associated by the relationship

(1+Nominal Yield) = (1+Real yield)(1+Expected inflation)(1+Effective risk premium)

This relationship can also be expressed as

(1+Nominal yield) = (1+Expected nominal yield of real security)(1+Effective risk premium) (3)

The effective risk premium in nominal yields arises because of inflation risk. This relationship assumes a market consensus on the measure of inflation to be used; the actual index used may be different. We can, however, view Equation (3) as a relationship between actual yields of nominal and inflation-indexed securities. If the risk embodied in the inflation index used is comparable to the risk of inflation embodied in nominal securities, then the risk premium calculated from Equation (3) should equal the market risk premium for fair valuation of inflation-indexed securities.

Risk Premium Estimated at 21-35 bp

We estimate the effective risk premium in the U.S. market based on our estimates of the risk premium for the U.K. market. The effective risk premium has two components: the pure risk premium due to inflation risk and the inflation convexity effect that counteracts the inflation risk premium. The risk premium due to an uncertain pay-off can be expressed as the market price of risk times the variance of the return of the pay-off.[4] Thus, the inflation risk premium and the convexity effect are both proportional to the variance of inflation, making the effective risk premium also proportional to the variance of inflation. The convexity of a 10-year Gilt is close to the convexity of a 10-year Treasury. Hence, under the assumption that the market price of inflation risk in the U.S. is the same as in the U.K., we can estimate the effective inflation risk premium in the U.S. market.

[4]For example, the excess return of the market portfolio in the Capital Asset Pricing Model equals the market price of risk times the variance of the portfolio's returns.

Effective U.S. risk premium =
Effective U.K. risk premium x (Variance of U.S. inflation/Variance of U.K. inflation) (4)

We estimated the average effective risk premium in the U.K. between May 1995 and October 1996 at 197 bp. During the same period, we estimate that the average standard deviation of a 12-month rolling series of monthly inflation changes in the U.S. was 38% of that in the U.K. Using Equation (4), this implies an effective risk premium in 10-year U.S. yields of 28 bp (=197 x $(0.38)^2$). If we have overestimated the U.K. risk premium because of underestimating market expectations of inflation, the estimate for the U.S. would be lower. For example, underestimating U.K. inflation expectations by 50 bp would imply an effective risk premium of 21 bp for the U.S.

These estimates of U.S. inflation risk premium should be viewed as educated guesses. Furthermore, the risk premium is not constant over time but will vary with the perceived volatility of future inflation. It makes sense to use a range for the risk premium when comparing nominal and inflation-indexed securities. Looking at October 1996, instead of considering averages for the entire period from May 1995 to October 1996, gives a slightly higher estimate for the risk premium. We estimate that the U.K. effective risk premium at the end of October 1996 was 164 bp, while the relative volatility of inflation over the preceding 12 months was 46%. This results in an estimate for the U.S. 10-year effective risk premium of 35 bp, or 25 bp with possible underestimation of expected U.K. inflation. We estimate the current effective risk premium in 10-year U.S. nominal yields, therefore, to be in a range of 21 bp to 35 bp.

VIII. RELATIVE PRICE PERFORMANCE

Relative Performance Will Depend on Economic Regime

The price performance of inflation-indexed securities compared to nominal securities will depend on the relative volatilities of their yields and the correlation between the two. These quantities, in turn, depend on the relative volatilities of real yields and the nominal to real yield spread, and the correlation between the two.

Change in nominal yield = Change in real yield + Change in nominal to real yield spread (5)

To understand the relative price performance of inflation-indexed versus nominal securities, we focused on the two components of nominal yields. These components reflect different economic fundamentals. As we saw from the U.K. experience, real yields are related to the state of the economy whereas the yield spread is related to monetary phenomena. The volatilities of the two components and the correlation between them will depend on the particular economic regime at the time.

As we saw in the U.K. experience, real yields tend to be higher in periods of higher economic growth and fall when economic growth falters. The volatility of real yields is, therefore, likely to be correlated with the flexibility of the real economy. Real yields may be more volatile in an economy that is able to adjust its outputs faster to external shocks than in a more rigid economy. Similarly, the yield spread may be more volatile in an economy with a more volatile inflation process. This could be due to a less stable monetary policy or the openness of the economy to external shocks.

The correlation between real yield and yield spread is likely to be a result of market expectations of the interaction of monetary and fiscal influences. When the economy is on a stable growth path—inflation is controlled and the market believes the central bank will remain vigilant adjusting monetary policy as needed to control inflation— the correlation between real yield and yield spread is likely to be zero. The Fisher hypothesis is likely to hold for long-term yields; Figure 14 shows this for the U.K. since September 1994. We believe the U.S. economy is currently in a similar situation, and we expect insignificant correlation between real yield and yield spread in the U.S. over the immediate future.

The correlation between real yield and yield spread could be negative when the markets anticipate an increase in economic growth and a lower future inflation rate. This could happen when a credible central bank eases monetary policy: lower rates would lead to higher growth while the fact that the central bank is lowering rates could itself lower expectations of future inflation. We see evidence of this in the U.K. in Figure 14. A negative correlation could also result when the central bank is tightening monetary policy aggressively. This could lead to expectations of a slower economy, but the central bank's actions could be interpreted as a signal that inflation is likely to be worse than previously anticipated, leading to higher inflation expectations. In the U.K., we found that correlations during such periods were negative but not statistically significant.

A positive correlation between real yield and yield spread would result when expectations of economic growth and inflation are revised in the same direction, for instance when the central bank is perceived to be inflating the economy to generate growth. Alternatively, if the central bank is perceived to be tightening monetary policy to contain future inflation, the market could revise upward its expectations of future growth and inflation, inducing a positive correlation between real yield and yield spread. While we do not see such an episode in the U.K., this may have been the situation in the U.S. in early 1994 when the Fed began its last series of tightenings.

The relative volatilities of and correlations between real yields and nominal yields are related to those of the real yield and yield spread through the following relationships.

Let

k = Standard deviation (Real yield change)/Standard deviation (Yield spread change)

q = Correlation (Real yield change, Yield spread change)

then

Standard deviation (Real yield change)/Standard deviation (Nominal yield change) = $1/[1+(1/k^2)+(2q/k)]^{1/2}$ (6)

Correlation (Real yield change, Nominal yield change) = (1+q/k) x Standard deviation (Real yield change)/ Standard deviation (Nominal yield change) (7)

We Estimate Relative Volatilities Around 80%

Our estimates for the relative volatilities and correlations of U.S. real and nominal 10-year yields for current economic conditions are shown in Figure 15. To estimate the ratio of volatilities of real yield and yield spread, we used the ratio in the U.K. between May 1995 and November 1996, a period when the fiscal and monetary regime in the U.K. was similar to the current regime in the U.S. The ratio in this period was 0.57. We adjusted this

Figure 15. **Projected Relative Volatilities and Correlations for U.S. 10-year Real and Nominal Yields**

	Case 1	Case 2
Std. deviation (Real yield change) / Std. deviation (Yield spread change)	1.24	1.50
Correlations (Real yield change, Yield spread change)	0.00	0.00
Std. deviation (Real yield change) / Std. deviation (Nominal yield change)	0.78	0.84
Correlation (Real yield change, Nominal yield change)*	0.78	0.84
Ratio of price volatilities	0.92	0.98

*Correlations and ratio of standard deviations are identical because correlation between real yield and yield spread is assumed to be zero.

ratio upward to reflect the lower volatility of inflation in the U.S. In Case 1, the adjustment was made using the relative volatility of inflation over the 12-month period ended in October 1996 (0.57/0.46 = 1.24); in Case 2, the adjustment used the average relative volatilities of 12-month rolling inflation over the entire period from May 1995 to October 1996 (0.57/0.38 = 1.50). Based on current stable growth and inflation outlook for the U.S. economy, we project the correlation between 10-year real yield and yield spread to be zero. With these assumptions, we project the relative volatilities of real and nominal yields to be between 0.78 and 0.84 and the relative price volatilities (taking into account the longer duration of the inflation-indexed securities) to be between 0.92 and 0.98. Our estimates depend on relative volatilities of inflation. Higher volatility of inflation in the U.S. would lead to lower relative volatility of real versus nominal yields. Given the current low volatility of inflation, we think that any change in relative volatilities in the future is likely to lower the ratio rather than increase it.

IX. DURATIONS AND HEDGE RATIOS

Real and Nominal Durations

The price sensitivity of inflation-indexed securities with respect to real yields is the duration of these securities for real yields, calculated in the usual manner. For example, the 10-year inflation-indexed security would have a duration of 8.38 years for real yields. (For simplicity, we are ignoring the effect of the two-month delay.) The question of duration related to nominal yields does not have a unique answer; it depends on the context in which inflation-indexed securities are deployed. Even for nominal Treasuries, comparing durations of two securities makes sense only when their yield changes are perfectly correlated and change in equal amounts. Practically speaking, since a major proportion of yield curve changes can be accounted for by parallel changes, duration becomes a meaningful measure of the risk of a portfolio. Duration of most spread products also has practical value because yield curve changes are normally far more volatile than spread changes. In the case of inflation-indexed securities, most of the conventional wisdom supporting the use of duration as a measure of risk is invalid. For example, in Case 1 in Figure 15, we expect the price volatility of the 10-year inflation-indexed Treasury to be 92% of the price volatility of the 10-year nominal. It is, however, inappropriate to equate the inflation-indexed security to a nominal Treasury with 92% of the duration of the 10-year nominal. This is because a substantial portion of the price volatility of the 10-year nominal Treasury is due to the 10-year inflation-related spread, which is independent of real yield volatility.

Nominal yields consist of real yields plus a spread related to future inflation. In this respect, when compared to inflation-indexed securities, nominal securities are like spread product. They have a duration related to real yields and a duration for the inflation-related spread just as a corporate bond has a duration related to the Treasury curve and a spread duration. In the present case, nominal Treasuries have a duration related to real yields and the inflation spread (the two are identical) while inflation-indexed Treasuries have a duration related to real yields, but none related to inflation spread. Substituting maturity-matched inflation-indexed securities for nominals increases the real duration of the portfolio while decreasing its inflation spread duration.

Hedge Ratio Depends on Investor Objective

In this situation, it is useful to think in terms of a hedge ratio based on a nominal security. The appropriate hedge ratio will depend on the exposure of the hedge to real yield and inflation spread, and the variance of the residual risk. In an environment where the volatility of nominal yields is entirely due to the inflation spread, the hedge ratio of 10-year TIPS with respect to the 10-year will be zero. On the other hand, if nominal yield volatility is entirely due to real yield volatility, the hedge ratio will be in the ratio of their respective durations.

In the current market environment neither of these extreme cases is likely. We expect 10-year real yields to be around 80% as volatile as nominal yields and the inflation spread to be uncorrelated with real yields. Even though it may seem reasonable to assume that relative to real yields inflation spread may be taken to be constant, it is erroneous to do so. In terms of variances (which is the appropriate measure of risk since variances are additive), real yield volatility accounts for only 64% of the variance while the inflation spread accounts for 36%. Since there is no perfect hedge available for the inflation spread component of nominal yields, any hedge of inflation-indexed securities by nominal securities will contain a substantial amount of residual risk.

For example, if the hedge ratio equals the ratio of durations of the 10-year inflation-indexed security for real yield and the 10-year nominal security for nominal yield (approximately 1.18), the hedge will have zero duration for real yields and -8.38 year duration for inflation spread. The standard deviation of the price return of this hedge will be 80% of the standard deviation of the price return of the inflation-indexed security for Case 1 in Figure 15 and 66% for Case 2.

Alternatively, a hedge ratio can be determined that minimizes the variance of the residual risk of the hedge. With a zero correlation between real yield and inflation spread, such a variance minimizing hedge ratio is given by:

Hedge ratio
= (Duration of inflation-indexed/Duration of nominal)
x (Variance of real yield/Variance of nominal yield) (8)

The minimum variance hedge ratios for the 10-year inflation-indexed versus the 10-year nominal Treasury are shown in Figure 16 for the two cases when the relative volatility of real to nominal yields is 0.78 and 0.84. The trade-off compared to using the ratio of durations is that the residual risk of the hedge is lower, but the hedge has a duration for real yields.

The appropriate hedge ratio depends on investor objective. From a portfolio perspective, it may be more desirable to have zero duration for real yields (i.e., a hedge ratio of 1.18) at the cost of greater volatility. From a trading perspective, the minimum variance hedge ratio may be more appropriate.

Figure 16. **Minimum Variance Hedge Ratio versus 10-year Nominal**

	Case 1	**Case 2**
Ratio of standard deviation of real to nominal yields	0.78	0.84
Minimum variance hedge ratio vs. 10-year nominal	0.72	0.83
Ratio of standard deviation of price returns of hedge to unhedged	0.62	0.55
Duration with respect to real yield	3.27 yr	2.52 yr
Duration with respect to inflation spread	-5.11 yr	-5.86 yr
Ratio of standard deviation of price returns when hedge is 118%	0.80	0.66

X. RELATIVE VALUE OF INFLATION-INDEXED TREASURIES IN A PORTFOLIO CONTEXT

In this section we focus on three types of portfolio investors: buy-and-hold fixed income investors whose long-term objective is to protect investment returns against inflation, total return fixed income investors whose performance is measured against a benchmark, and total return investors whose portfolios include equities and fixed income instruments. Each type of investor is likely to find inflation-indexed securities attractive for different reasons. We think inflation-indexed securities can improve the risk-return profile of the portfolios of all three types of investors.

Relative Value of TIPS Depends on Their Expected Nominal Yield

In comparing the relative value of inflation-indexed securities, we think investors need to form a view on expected inflation. The expected nominal yield of TIPS can then be compared with the yield of nominal securities or used to estimate the expected total return of these securities over a holding period. Because inflation expectations may vary across investors, there is likely to be a diversity of opinions on relative value. We think this is unavoidable. Currently, market talk seems to center on a long-term inflation forecast of 3%. While this is reflected in some market surveys such as the Philadelphia Fed's survey, expectations of long-term inflation will remain subjective. We compared the relative value of 10-year TIPS by evaluating the nominal yield give-up compared to the 10-year at which investors are likely to find them attractive.

Attractive Inflation Hedge for Buy-and-hold Investors

From the perspective of buy-and-hold investors whose objective is to protect investment returns from inflation, the advent of inflation-indexed securities provides a benchmark for evaluating other investments. Over their maturity, inflation-indexed securities provide a risk-free return after adjusting for inflation (ignoring the reinvestment risk of the coupon flows), making nominal Treasuries the risky asset. Nominal Treasuries should be evaluated on the basis of the extra expected real return they offer compared to the risk of inflation.

Ignoring compounding effects, the excess expected real return of nominal Treasuries is given by:

Required risk premium
= Excess expected real return of nominal Treasuries
= Nominal yield of nominal securities - Expected inflation
+ Inflation convexity effect - Real yield of inflation-indexed security.

By grouping expected inflation with the real yield of the inflation-indexed security, this can also be expressed as

Required risk premium
= Nominal yield of nominal securities
+ Inflation convexity effect - Expected nominal yield of inflation-indexed security (9)

This excess expected real return must be compared with the inflation risk of nominal securities. To estimate the current risk of inflation in 10-year nominal Treasuries, we fitted the time series of CPI since 1989 with a first order autoregressive process, and then used the estimated statistical parameters to simulate the probability distribution of the CPI over the next 10 years. Although the expected value of inflation may differ across investors, we think our risk estimates provide a measure for long-term inflation risk. Assuming the CPI continues to follow the same statistical process as its recent history, we project the mean rate of CPI inflation over the next 10 years to be 3.46%. The one standard deviation path of CPI around this projected mean is 0.38% at an annualized rate. We estimate the inflation convexity effect at this volatility to be 1 bp to 2 bp. Our results mean that there is a 16% probability that the realized real return of a 10-year nominal Treasury could be lower by more than an annualized 36 bp than its expected real return (again ignoring intervening cash flows and inflation). Looked at another way, if the expected excess real return of nominal Treasuries is 36 bp, there is a 16% probability that the realized real return of nominal Treasuries would be lower than the real return of inflation-indexed Treasuries. Although the required risk premium is likely to vary across investors, we estimate the current market inflation risk premium and convexity effect for 10-year nominal Treasuries to be 21-35 bp. With the risk premium in this range, from a buy-and-hold investor's perspective, inflation-indexed securities would be cheap to fairly valued.

Diversification Benefits for Fixed Income Total Return Investors

Total return-oriented fixed income investors are a diverse group, and they are unlikely to evaluate the attractiveness of inflation-indexed Treasuries using uniform criteria. We evaluated these securities by estimating how the inclusion of inflation-indexed securities in a portfolio of nominal Treasuries would improve the portfolio's risk-return trade-off in a mean-variance framework. We did this for two cases of relative volatility of real and nominal 10-year yields (see Figure 15). For each case, we evaluated the attractiveness of 10-year inflation-indexed Treasuries at nominal yield give-ups of 22 bp, 17 bp, and 12 bp compared to the 10-year on-the-run in a portfolio that consisted of 3-, 5-, and 10-year nominal Treasuries.

The mean-variance efficient portfolios were constructed for a three-month horizon. Expected total returns for the securities were calculated assuming unchanged yield curve and including the rolldown and convexity effect. The rolldown effect for the 10-year inflation-indexed Treasury was assumed to be 80% of that for the 10-year nominal Treasury. Volatilities and correlations for the nominal Treasuries were estimated from weekly data for the period May 1995 to November 1996. Yield volatility for the 10-year inflation-indexed security was derived from the relationships in Figure 15. The covariance of the 10-year real yield with nominal yields was derived using the relationships observed in the U.K. (see Appendix B). For the case when the relative volatility of 10-year real yields is 84% of that of nominal yields (Case 2 in Figure 15), Figure 17 shows the correlation matrix and Figure 18 shows the yield volatilities used in the analysis.

We examined the efficient portfolios for two risk levels; a quarterly standard deviation of 2.5%, which corresponds to the price volatility of the 5-year, and a higher quarterly standard deviation of 3.25%. The portfolio compositions for the cases we examined are shown in Figure 19. We found that at a 0.78 relative yield volatility, 10-year inflation-indexed Treasuries improve the mean-variance efficiency of a portfolio of nominal Treasuries that has a 2.5% standard deviation of price returns at a yield give-up of 22 bp compared to the 10-year nominal Treasury. The mean-variance efficient portfolio contains 2% in the 10-year inflation-indexed security, 96% in the 5-year, and 2% in the 3-year. For a portfolio with a 3.25% standard deviation of returns, the 10-year inflation-indexed enters the portfolio only when the yield give-up is 17 bp. If the relative volatility of real to nominal 10-year yields is 0.84, then the 10-year inflation-indexed security improves the mean-variance efficiency of a portfolio only when the yield give-up compared to the 10-year nominal Treasury is 12 bp.

Our analysis indicates that 10-year inflation-indexed Treasuries have the potential to improve the risk-return profile of a portfolio of nominal Treasuries at yield give-ups of 22 bp to 12 bp relative to the 10-year nominal Treasury. The improvement occurs because the 10-year inflation-indexed security, which is substituted for the 10-year nominal, has better diversification characteristics. The yield give-up depends crucially on the relative volatility of real to nominal yields. A small decrease in the relative volatility makes inflation-indexed securities more attractive at a particular yield give-up. We think that current economic conditions are especially biased toward producing a high relative volatility of real to nominal yields. Going forward, relative volatility is more likely to decrease than increase. In that case, inflation-indexed securities are likely to outperform nominal Treasuries as a higher yield give-up can be justified based on the lower relative volatility and the diversification benefit. Finally, the higher end of our yield give-up falls in the range of our estimated risk premium for the 10-year U.S. sector. Thus, at a suitable yield give-up, both buy-and-hold and total return investors could be drawn to the inflation-indexed market and increase its liquidity.

Efficient Hedge for Equity Investors

Inflation-indexed securities are also likely to be attractive for portfolios consisting of equities and fixed income securities. Over a reasonable holding period, total returns for equities are likely to be positively correlated with inflation and economic growth, while fixed coupon nominal securities are negatively correlated. Inflation-indexed securities should fall between these two extremes. The returns will be positively correlated with

Figure 17. **Assumed Correlations of Yield Changes of Nominal and Inflation-indexed Treasuries**
Assuming relative volatility of real and nominal 10-year yields of 0.84

	Nominal				Inflation-indexed
Nominal	**3-yr**	**5-yr**	**10-yr**	**30-yr**	**10-yr**
3-yr	1.00	0.99	0.97	0.93	0.76
5-yr		1.00	0.99	0.96	0.77
10-yr			1.00	0.99	0.84
30-yr				1.00	0.85
Inflation-indexed 10-yr					1.00

Figure 18. **Assumed Yield Volatilities**
Annualized, assuming relative volatility of real and nominal 10-year yields of 0.84

Sector	Yield Volatilities
3-yr	121 bp
5-yr	120
10-yr	109
30-yr	90
Inflation-indexed 10-yr	91

Figure 19. **Mean-variance Efficient Portfolios**
Market-value weights, %

Rel. Vol. of 10-yr. Real to Nom. Ylds.	Std. Dev. of Port. Return	Nominal Yield Spread of 10-year Inflation-indexed to 10-year On-the-run: -22 bp				-17 bp				-12 bp			
		3-yr	5-yr	10-yr	10-yr infla.-indexed	3-yr	5-yr	10-yr	10-yr infla.-indexed	3-yr	5-yr	10-yr	10-yr infla.-indexed
0.78	2.50%	2	96	-	2	3	88	-	9	5	79	-	16
	3.25%	-	43	57	-	-	40	56	4	-	32	53	15
0.84	2.50%	7	89	4	-	7	89	4	-	5	87	-	8
	3.25%	-	43	57	-	-	43	57	-	42	56	-	2

inflation because of the inflation protection but negatively correlated with economic growth because real yields are likely to rise when economic growth picks up. Although equity returns could also be negatively affected by higher real yields, this is more than compensated for by the higher expected earnings resulting from higher economic growth. Inflation-indexed securities could, therefore, be a better hedge than nominal fixed coupon securities against the risk to an equity portfolio of a decline in the economy. Nominal securities are exposed to the combined effect of inflation and economic growth while inflation-indexed securities are exposed only to the risk of economic growth.

XI. CHOICE OF INFLATION INDEX

Any measure used to index inflation-protected securities is unlikely to be the appropriate measure of inflation for all investors or the "true" inflation measure incorporated in the nominal yield curve. Furthermore, the inflation measure incorporated in nominal securities may be changing over time, depending on investor demand. One method of comparing inflation-indexed and nominal securities would be to compare expected nominal returns based on projection of the specific inflation index used for inflation-indexed securities. Investors could then determine whether the nominal returns are commensurate with the reduction in inflation risk afforded.

The major factor in this comparison is likely to be differences between expected values of an investor's preferred inflation index and the index used for inflation-indexed securities. However, the risk introduced by the relative volatilities of different inflation indices may not be significant. For example, since 1987 inflation as measured by the CPI has been approximately 54 bp per year higher than as measured by the chain-linked GDP deflator. This difference has been attributed to the substitution effect, which is better accounted for in the GDP deflator. The deviation of each series from its trend line is, however, not significant. It is likely that, to the first order of magnitude, any future inflation shocks will affect different inflation indices similarly.

The issues raised by the recent Boskin Commission report on the method of measuring inflation are, in principle, similar to the issue of choosing the appropriate inflation index. The effect on inflation-indexed securities of a change in the way the CPI is computed is not easy to estimate. If the proposed changes are driven by a concern for measuring true inflation, not by an attempt at artificially lowering measured inflation, then the securities will be even more effective in protecting purchasing power. As the new method is scrutinized and becomes credible, the market may adjust the compensation for long-term inflation even in nominal securities, leading to a gradual lowering of nominal yields. The relevant question for an investor over a medium-term horizon is to compare returns of inflation-indexed securities with inflation-adjusted returns of nominal securities, taking into account the reduction in inflation risk. Of course, there may be short-term price volatilities.

XII. CONCLUSION

Inflation-indexed securities will be a useful addition to the investment choices available to buy-and-hold as well as total return fixed income investors. The securities are also likely to be beneficial to equity investors. These securities may initially experience some excess price volatility as the market becomes familiar with the pricing implications of some of their structural features. We expect liquidity to improve as both buy-and-hold and total return investors enter the market and other institutional features such as the repo market and coupon stripping develop to support secondary trading.

Appendix A. **Computation of Effective Real Yields for TIPS**

For a security to be evaluated at time *t*, with *2T* coupon payments remaining to maturity, we have

$$
\text{Present Value}_t =
\begin{cases}
\text{Case (a)} \\
\displaystyle\sum_{i=0}^{2T-1} \frac{\frac{c}{2} \bullet \left(\frac{\text{CPI}_t}{\text{CPI}_{base}}\right) \Big/ \left(1+f^n_{\left(\frac{i}{2}+\tau-del\right),\left(\frac{i}{2}+\tau\right)}\right)}{\left(1+\frac{y_2}{2}\right)^{2\bullet\left(\frac{i}{2}+\tau-del\right)}} \\
+\displaystyle\frac{100 \bullet \frac{\text{CPI}_t}{\text{CPI}_{base}} \Big/ \left(1+f^n_{\left(T-\frac{1}{2}+\tau-del\right),\left(T-\frac{1}{2}+\tau\right)}\right)}{\left(1+\frac{y_r}{2}\right)^{2\bullet\left(T-\frac{1}{2}+\tau-del\right)}} \\
\text{if } \tau > del \\
\\
\text{Case (b)} \\
\displaystyle\frac{c}{2} \bullet \frac{\text{CPI}_{t-(del-\tau)}}{\text{CPI}_{base}} \Big/ (1+f^n_{0,\tau}) \\
+\displaystyle\sum_{i=1}^{2T-1} \frac{\frac{c}{2} \bullet \left(\frac{\text{CPI}_t}{\text{CPI}_{base}}\right) \Big/ \left(1+f^n_{\left(\frac{i}{2}+\tau-del\right),\left(\frac{i}{2}+\tau\right)}\right)}{\left(1+\frac{y_r}{2}\right)^{2\bullet\left(\frac{i}{2}+\tau-del\right)}} \\
+\displaystyle\frac{100 \bullet \frac{\text{CPI}_t}{\text{CPI}_{base}} \Big/ \left(1+f^n_{\left(T-\frac{1}{2}+\tau-del\right),\left(T-\frac{1}{2}+\tau\right)}\right)}{\left(1+\frac{y_r}{2}\right)^{2\bullet\left(T-\frac{1}{2}+\tau-del\right)}} \\
\text{if } \tau \le del
\end{cases}
$$

where

y_r =	real yield
c =	real coupon rate
CPI_t =	Consumer Price Index at time *t*
CPI_{base} =	Consumer Price Index applicable at issuance
T =	number of years remaining to maturity, rounded up to the closest half year
τ =	period remaining to the next coupon
del =	delay period (= 2 months = 1/6 years in the U.S.)
$f^n_{a,b}$ =	unannualized nominal forward rate applicable to the time period starting at a and ending at b.

Case (a) covers the possibility that the next coupon coming due will be paid more than 2 months from time *t*.

Case (b) covers the possibility that this coupon comes due within 2 months from time *t*. This coupon is then unindexed to inflation due to the 2 months delay, and it becomes a completely nominal payment. In the U.K., where the delay is 7-8 months and coupons are paid on a semiannual basis, the first upcoming coupon is always fixed in nominal terms. The second coupon is unindexed during the last 2 months before payment.

Appendix B. **Construction of the U.S. Covariance Matrix including 10-year TIPS**

To solve the mean variance optimization problem described in the text, we need the covariance matrix of price returns. This was obtained by multiplying the elements of the covariance matrix of yield changes with the appropriate durations. We estimated the covariance matrix of yield changes for the U.S. as follows:

For the 4 by 4 submatrix containing the 3-, 5-, 10-, and 30-year nominal bonds, we used the empirical covariance matrix estimated from U.S. data. This estimate is based on weekly data for the period May 1995 to November 1996.

Because real rates and spreads add to nominal rates, the covariance of the 10-year index-linked bond with a T-year maturity nominal bond equals the sum of the covariances of the T-year spread with the 10-year real rate and the T-year real rate with the 10-year real rate. Given the covariances between the 10-year real yield and the 3-, 5-, 10-, and 30-year real yield and the 3-, 5-, 10-, and 30-year spread, we can find the covariance of 10-year real yields with 3-, 5-, 10-, and 30-year nominal yields. This requires a covariance matrix between spreads and real rates for the U.S. This was computed in two steps:

First, the volatilities of the spreads and the real rates for the 3-, 5-, 10-, and 30-year maturities were derived. For each country, we define the volatility ratio as

Volatility ratio = Real rate volatility/spread volatility

We then assume that

Volatility ratio in U.K./Volatility ratio in U.S.
= Inflation volatility in U.S./ Inflation volatility in U.K.

The greater a countries' inflation volatility, the more volatile its spread was assumed to be compared to its real rate. Using this relationship, we could derive spread and real rate volatilities for the U.S. from the volatilities of the same maturity nominal rates.

Second, for the correlation matrix of spreads and real rates, we used the U.K. correlation matrix, based on weekly data covering May 1995 to November 1996. In all calculations, if an estimated correlation was not significantly different from zero at the 95% level, we set it to zero.

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